Normalizations on CHEASE

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1 CHEASE normalizations, MKSA units

From

$$\underline{B} = T\underline{\nabla}\phi + \underline{\nabla}\phi \times \underline{\nabla}\psi \tag{1}$$

then

$$[B] = \frac{[T]}{[R]} \rightarrow T_{\text{CHEASE}} = \frac{T_{\text{phys}}}{B_0 R_0} \tag{2}$$

Where R_0 and B_0 are the values used for normalizing the parameters

$$B_{0_{\text{CHEASE}}} = B_{0_{\text{phys}}}/B_0 \qquad R_{0_{\text{CHEASE}}} = R_{0_{\text{phys}}}/R_0 \tag{3}$$

If one uses the boundary conditions $T_{\text{CHEASE}}(\text{edge}) = 1$, then B_0 is the vacuum magnetic field at $R = R_0$ such that $T(\text{edge}) = R_0 B_0$. From

$$\underline{\nabla} \times B = \mu_0 j \tag{4}$$

we obtain the following expression

$$[j] = \frac{[B]}{\mu_0\,[R]} = \frac{[B]}{\mu_0\,[R]} \tag{5}$$

which gives the following normalization for the current density:

$$j_{\text{CHEASE}} = j_{\text{phys}} \frac{\mu_0 R_0}{B_0} \tag{6}$$

and the corresponding relation for total current

$$[I] = \frac{[B]\,[R]}{\mu_0} \rightarrow I_{\text{CHEASE}} = I_{\text{phys}} \frac{\mu_0}{R_0 B_0} \tag{7}$$

For the pressure, from

$$\nabla p = j \times B \tag{8}$$

we obtain

$$[p] = [j]\,[B]\,[R] = \frac{B_0}{\mu_0 R_0} B_0 R_0 = \frac{B_0^2}{\mu_0} \tag{9}$$

Then we define

$$p_{\text{CHEASE}} = p_{\text{phys}} \frac{\mu_0}{B_0^2} \tag{10}$$

The magnetic field itself is defined by Eq. 1 which brings to

$$[\psi] = [B]\,[R]^2 \tag{11}$$

It follows

$$\psi_{\text{CHEASE}} = \frac{\psi_{\text{phys}}}{B_0 R_0^2} \tag{12}$$

In the same way

$$p' = \left[\frac{\partial p}{\partial \psi}\right] = \frac{[p]}{[\psi]} = \frac{B_0^2}{\mu_0 R_0^2 B_0} = \frac{B_0}{\mu_0 R_0^2} \to p'_{\text{CHEASE}} = p'_{\text{phys}} \frac{\mu_0 R_0^2}{B_0} \tag{13}$$

and

$$TT' = \left[T \frac{\partial T}{\partial \psi}\right] = \frac{[T]^2}{[\psi]} = \frac{B_0^2 R_0^2}{B_0 R_0^2} = B_0 \to TT'_{\text{CHEASE}} = \frac{TT'_{\text{phys}}}{B_0} \tag{14}$$

Let us now check β_{pol} as given in Table 1 of H. Lutjens et al, Comput. Phys. Commun. 97 (1996) 219:

$$\beta_{\text{pol}} = \left[\frac{8\pi}{I_\phi^2 R_0} \overline{p} V_{tot}\right]_{\text{CHEASE}} \tag{15}$$

Note that $V_{tot,\text{CHEASE}}$ is in fact $V_{\text{CHEASE}}/(2\pi)$, thus we get after the above transformations:

$$\beta_{\text{pol}} = \frac{4}{\mu_0} \frac{\int p_{\text{phys}} dV_{\text{phys}}}{I_{\phi_{\text{phys}}}^2 R_{0_{\text{phys}}}} \tag{16}$$

which corresponds to the usual definition with $\overline{B_{\text{pol}}} = \mu_0 I_{\phi_{\text{phys}}} / (2V/R_{0,\text{phys}})^{1/2}$

Let us now focus on the parallel component of the plasma current as given in Eq. (8) of the above paper. We get:

$$\left[I_\parallel\right] = \frac{[j]}{[1/R]} = [j]\,[R] \tag{17}$$

Instead of $[j]$. This is because a $1/R_0$ is missing in Eq. (8). R_0 is supposed to be equal to 1 in CHEASE, so it's not so important, however the correct definition should read and has been modified in CHEASE:

$$I_\parallel = \frac{<\mathbf{j}\cdot\mathbf{B}>}{R_0 <\mathbf{B}\cdot\nabla\phi>} \tag{18}$$

In this way, $I_\parallel$ has the correct dimension and we have:

$$I_\parallel = [j] \Rightarrow I_{\parallel_{\mathrm{CHEASE}}} = I_{\parallel_{\mathrm{phys}}}\frac{\mu_0 R_0}{B_0} \tag{19}$$

2 CHEASE normalizations in CGS

In CGS

$$\underline{\nabla}\times B = \frac{4\pi j}{c} + \frac{1}{c}\frac{\partial E}{\partial t} \tag{20}$$

$$mn\frac{dv}{dt} = qnE + \frac{qnv}{c}\times B - \underline{\nabla}p \tag{21}$$

$$\Rightarrow \frac{j}{c}\times B = \underline{\nabla}p \tag{22}$$

$$\underline{B} = T\underline{\nabla}\phi + \underline{\nabla}\phi\times\underline{\nabla}\psi \tag{23}$$

Dimensionally, again

$$[T] = [B_0]\,[R_0] \tag{24}$$

where this time B_0 [Gauss] and R_0 [cm]. The latter relations thus bring to

$$[j]_{\mathrm{CGS}} = \frac{c}{4\pi}\frac{[B]}{[R]} = \frac{c}{4\pi}\frac{[B_0]}{[R_0]} \tag{25}$$

$$[p]_{\mathrm{CGS}} = \frac{[j]\,[B]}{c}\,[R] = \frac{c}{4\pi}\frac{B_0}{R_0}\frac{1}{c}B_0R_0 = \frac{B_0^2}{4\pi} \tag{26}$$

$$[\psi] = [B]\,[R]^2 = B_0R_0^2 \tag{27}$$

$$\Rightarrow [p']_{\mathrm{CGS}} = \frac{B_0^2}{4\pi B_0 R_0^2} = \frac{B_0}{4\pi R_0^2}$$

$$\Rightarrow p'_{\mathrm{CHEASE}} = p'_{\mathrm{CGS}}\frac{4\pi R_{0_{\mathrm{CGS}}}^2}{B_{0_{\mathrm{CGS}}}} \tag{28}$$

Finally we can express TT' from these relations as

$$TT'_{\mathrm{CHEASE}} = \frac{TT'_{\mathrm{CGS}}}{B_{0_{\mathrm{CGS}}}} \tag{29}$$

Note that the following identity holds

$$\begin{aligned} [p]_{\mathrm{MKS}} &= \frac{B^2_{0_{\mathrm{MKS}}}}{\mu_0} = \frac{B^2_{0_{\mathrm{MKS}}}}{4\pi} 10^7 \\ [p]_{\mathrm{CGS}} = \frac{B^2_{0_{\mathrm{CGS}}}}{4\pi} &= \frac{B^2_{0_{\mathrm{MKS}}}}{4\pi} 10^8 \Rightarrow [p]_{\mathrm{CGS}} = 10\,[p]_{\mathrm{MKS}} \end{aligned} \tag{30}$$

which is the conversion factor in the NRL plasma formulary.