Tokamak Equilibrium Coordinate Conventions: *COCOS*[*]

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Abstract

The Grad-Shafranov axisymmetric equilibrium solution for tokamak plasmas, ψ, does not depend on the sign of the plasma current I_p nor of the magnetic field B_0. In addition, the sign, amplitude and shift of ψ is not so important either, since the free sources depend on the normalized radial coordinate. On the other hand, $dp/d\psi$ and $dF^2/d\psi$, with $F = RB_\varphi$, need to be consistent to provide the correct current density profile. Moreover, RF and CD codes (Radio Frequency heating and Current Drive) need to know the exact sign convention and to take into account the effective sign of I_p and B_0 in order to calculate the co- or counter-CD component for example. As is shown in this paper, there are at least 16 different cases and a new index *COCOS* is proposed to uniquely identify the coordinate conventions assumed. Given the present worldwide efforts for codes integration, the proposed new index *COCOS* defining uniquely the COordinate COnventionS required as input by a given code or module is very useful. Since different codes use different conventions, equilibrium codes should be able to have a specific convention as input and another convention as output of the code. In addition, given two different conventions, it is relatively easy to transform from one to another. The relevant transformations are described in detail as well.

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I. INTRODUCTION

The effective solution to the Grad-Shafranov equation [1–3] does not depend on the sign of the plasma current I_p nor on the sign of the magnetic field B_0, nor does the ideal MHD stability in axisymmetric plasmas (no dependence on sign of toroidal mode number for example) [4]. In general, axisymmetric tokamak equilibrium codes actually work in normalized units (like R_0 and B_0 for CHEASE [5]), which means that I_p and B_0 are always assumed to be positive. Moreover, many codes also assume q, the safety factor, to be positive although this is not necessarily the case. The relative signs depend on several choices:

1. The choice of the "cylindrical" coordinate system representing the tokamak, the direction of the toroidal angle φ and if the right-handed system is (R, φ, Z) or (R, Z, φ) (R is assumed to be always directed outwards radially and Z upwards). The sign of I_p and B_0 in this system is also important.

2. The choice of the orientation of the coordinate system in the poloidal plane. Mainly whether the poloidal angle is clockwise or counter-clockwise and whether $(\rho/\psi, \theta, \varphi)$ is right-handed or if it is $(\rho/\psi, \varphi, \theta)$. In addition, whether φ in the poloidal coordinate system has the same direction as the one in the cylindrical one. In this paper, we assume it is always the case.

3. The sign of $\psi \sim \pm \int \mathbf{B} \cdot d\mathbf{S_p}$.

In this work, we refer to the view from the top of the tokamak to determine the toroidal direction and, for the poloidal plane, looking at the poloidal cross-section at the right of the major vertical axis ($R = 0$). In this way, a plasma current flowing counter-clockwise in the toroidal direction (as seen from the top) leads to a poloidal magnetic field clockwise in the poloidal plane. Since the usual "positive" mathematical direction for angles is "counter-clockwise", one sees that there is a difficulty either for the toroidal angle or for the poloidal angle if one wants to follow the magnetic field line with the coordinate systems. This is the main reason why there are many choices for the coordinate systems. It should be noted that the sign of q depends on this choice as well. In the examples given below, and if not stated otherwise, the sign of q refers to the case where both I_p and B_0 are positive in the respective coordinate system.

Three examples are shown in Fig. 1 to illustrate how this "difficulty" has been resolved:

Fig. 1(a) In the CHEASE code [5] (and in Hinton-Hazeltine [6], ONETWO [7] for example), since the main plane for an axisymmetric toroidal equilibrium is the poloidal plane, it was chosen to have θ in the "positive" direction and the "natural" system (ρ, θ, φ) right-handed, and to have q positive with I_p and B_0 positive. Therefore φ has to be in the "negative" direction yielding (R, Z, φ) right-handed.

Fig. 1(b) Both θ and φ are kept in the geometrical "positive" direction (counter-clockwise). In this case q is negative (with I_p, B_0 in the same direction) and the right-handed poloidal system becomes (ρ, φ, θ) in order to have the same φ direction in both systems. This was chosen in Freidberg ([4]) and by the EU-ITM [8] up to 2011 for example, as well as in http://www-fusion.ciemat.es/fusionwiki/index.php/Toroidal_coordinates.

Fig. 1(c) The cylindrical system is chosen to be the conventional one and then θ is chosen such that q is positive while keeping the conventional right-handed system: (ρ, θ, φ). This leads to have θ clockwise. This is standard for Boozer coordinates [9] and was chosen for ITER [10].

There is no unique solution nor a "correct" solution. However the present authors think that the third option is the less prone to errors since it keeps the conventional right-handed orientations; it takes the usual choice for the right-handed cylindrical coordinate system (R, φ, Z); and it has q positive when both I_p and B_0 have same sign. Therefore vector calculus can be used with the conventional rules. This is probably why it is often the one used for 3D calculations. Note that it is foreseen to be the ITER convention [10]. In this paper, we first propose in Sect. II the new identifier *COCOS* which uniquely defines the COordinate COnventionS used by a code or set of equations for both the cylindrical and poloidal systems. In addition it defines the sign of the poloidal flux and if it is divided by 2π or not. We then derive the transformation to a given choice of coordinate systems in Sects. III and IV for a given reference equilibrium solution ψ_{ref}. In Sec. V we discuss how one can check the consistency of a *COCOS* equilibrium and how to determine the *COCOS* value used by a code or a set of equations. In Sec. VI we provide, with Appendix C, the general transformations from any *cocos_in* value to any *cocos_out* value, including the discussion of the normalizations and how to only change the sign of I_p and/or B_0. We then discuss differences between various *COCOS* choices (Sect. VII) and derive the

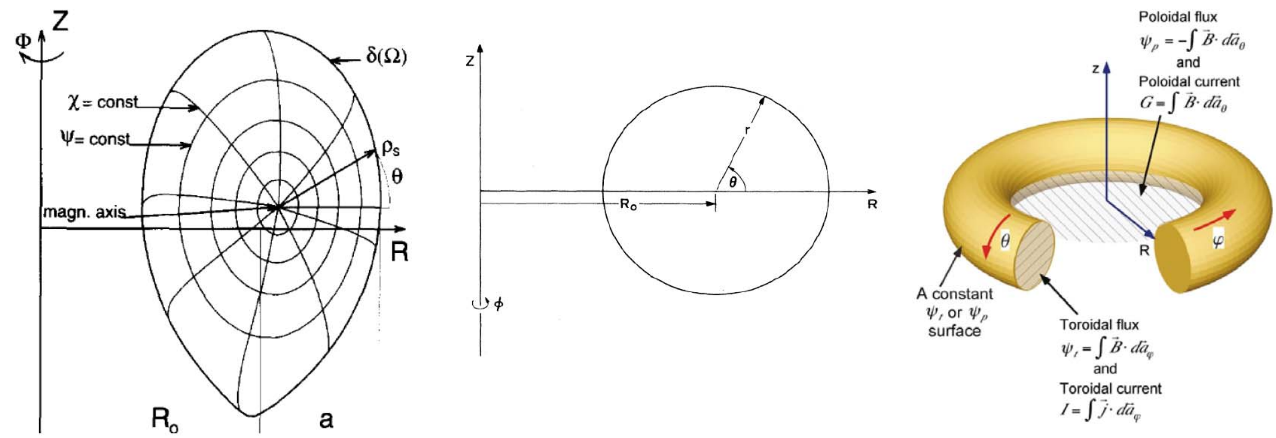

FIG. 1: Examples of cylindrical and poloidal coordinate systems: (a) CHEASE ([5], Fig. 1): $(R, Z, \varphi))$; (ρ, θ, φ), also used in [6]. (b) As in Freidberg ([4], Fig. 15): $(R, \varphi), Z)$; (ρ, φ, θ). (c) As in Boozer ([9], Fig. 1): (R, φ, Z) ; (ρ, θ, φ).

corresponding generic Grad-Shafranov equation to show how the equilibrium sources should be transformed as well (Sect. VIII). Conclusions are provided in Sect. IX.

II. *COCOS* INDEX: GENERIC DEFINITION OF B AND RELATED QUANTITIES

In order to stay general we can write the magnetic field $\mathbf{B}$ as follows:

$$\mathbf{B} = F\ \nabla\varphi + \sigma_{Bp}\ \frac{1}{(2\pi)^{e_{Bp}}}\ \nabla\varphi \times \nabla\psi_{ref}. \tag{1}$$

Using the standard (ρ, θ, φ) with ψ_{ref} increasing with minor radius, and with $sign(\mathbf{B}\cdot\nabla\theta) = sign(\partial\psi/\partial\rho)$, leads to $\sigma_{Bp} = 1$; and using (ρ, φ, θ) with ψ_{ref} decreasing with minor radius yields $\sigma_{Bp} = -1$. In addition, the poloidal flux ψ_{ref} can be chosen as the effective poloidal flux, yielding $e_{Bp} = 1$, or to the poloidal flux divided by 2π, in which case the exponent is zero: $e_{Bp} = 0$. The poloidal flux Ψ_{pol} is thus defined by:

$$\Psi_{pol} = -\sigma_{Bp} \int \mathbf{B}\cdot d\mathbf{S_p}, \tag{2}$$

with $\mathbf{dS_p}$ in the direction of a magnetic field at the major vertical axis that would be driven by a positive current in the relative φ direction. Note that it is in the direction of θ near the major axis with (ρ, θ, φ) right-handed and opposite with (ρ, θ, φ) left-handed. In this way,

$\mathbf{dS_p}$ can be defined for any (R_b, Z_b) point as the disc $R \leq R_b, Z = Z_b$ and the orientation just mentioned. This allows Ψ_{pol} to be well defined outside the last closed flux surface (LCFS), across the LCFS and also on the low field-side (LFS) of the LCFS. It leads to:

$$\psi_{ref} = -\sigma_{Bp} \frac{1}{(2\pi)^{(1-e_{Bp})}} \int \mathbf{B} \cdot d\mathbf{S_p}. \tag{3}$$

The minus sign expresses that the poloidal flux, in the standard right-handed system with $\sigma_{Bp} = +1$, is minimum at the magnetic axis and maximum otherwise. This is important since for example $dp/d\psi$ is then negative as expected for an increasing ψ "radial" coordinate. Coordinate systems which have $\sigma_{Bp} = -1$, that is $B_p = \nabla\psi_{ref} \times \nabla\varphi$, have ψ maximum at the magnetic axis and thus $dp/d\psi$ positive (when I_p is positive). Eq. (3) also shows that $e_{Bp} = 0$ when the poloidal flux ψ_{ref} is already divided by 2π and $e_{Bp} = 1$ when it is not.

Another way to refer to the poloidal flux definition is through the vector potential $\mathbf{A}$, in particular the φ component which is related to the poloidal magnetic field:

$$\mathbf{A}_\varphi = -\sigma_{Bp} \frac{\psi_{ref}}{(2\pi)^{e_{Bp}}} \nabla\varphi \Rightarrow A_\varphi = -\sigma_{Bp} \frac{\psi_{ref}}{(2\pi)^{e_{Bp}} R}, \tag{4}$$

which yields:

$$\mathbf{B_p} = \nabla \times \mathbf{A}_\varphi = \nabla \times \left(-\sigma_{Bp} \frac{\psi_{ref}}{(2\pi)^{e_{Bp}}} \nabla\varphi\right) = \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}} \nabla\varphi \times \nabla\psi_{ref}, \tag{5}$$

as defined in Eq. (1).

The toroidal flux is given by:

$$\Phi_{tor} = \int \mathbf{B} \cdot d\mathbf{S}_\varphi = \int B_\varphi dS_\varphi, \tag{6}$$

where $\mathbf{dS}_\varphi$ is the poloidal cross-section inside of the specific flux surface $\psi = \text{const}$ in the direction of the respective φ. Therefore it is always increasing with minor radius for positive B_0.

The general definition of q is given by the relative increase in toroidal angle per poloidal angle, or in other words the number of toroidal turns for one poloidal turn made by the equilibrium magnetic field line. It can be written as:

$$q = \frac{1}{2\pi} \int \frac{\mathbf{B} \cdot \nabla\varphi}{\mathbf{B} \cdot \nabla\theta} d\theta = \frac{\sigma_{Bp}}{(2\pi)^{1-e_{Bp}}} \int \frac{F}{R^2} J \, d\theta, \tag{7}$$

where we have introduced Eq. (1) and used $J^{-1} = (\nabla\varphi \times \nabla\psi_{ref}) \cdot \nabla\theta$ corresponding to the Jacobian of $(\psi_{ref}, \theta, \varphi)$. Defining $\sigma_{\rho\theta\varphi} = 1$ when (ρ, θ, φ) is right-handed and -1 when it is left-handed, and taking into account that ψ_{ref} increases/decreases with ρ for $\sigma_{Bp} = \pm 1$, we obtain:

$$q = \frac{\sigma_{\rho\theta\varphi}}{(2\pi)^{1-e_{Bp}}} \int \frac{F}{R} \frac{dl_p}{|\nabla \psi_{ref}|}. \tag{8}$$

Using Eq. (6) with $B_\varphi = F/R$ and $dS_\varphi = \sigma_{Bp} \frac{d\psi_{ref}}{|\nabla \psi_{ref}|} dl_p$, with the σ_{Bp} factor reflecting the fact that the toroidal flux is chosen to increase with minor radius ($\sigma_{Bp}\, d\psi_{ref} > 0$) and dl_p being the arc length of the magnetic surface contour in the poloidal cross-section, we get:

$$q = \frac{\sigma_{Bp}\, \sigma_{\rho\theta\varphi}}{(2\pi)^{(1-e_{Bp})}} \frac{d\Phi_{tor}}{d\psi_{ref}}. \tag{9}$$

Note that sometimes Φ_{tor} is divided by 2π in Eq. (6) such as to avoid the 2π in Eq. (9). It is important to note that the sign of q is positive or negative depending on the orientation of the poloidal coordinate system. This is recovered by Eq. (9) since $\sigma_{Bp}\, d\psi_{ref}$ is always positive. In many cases q is assumed always positive by some codes, even if $I_p < 0$ and $B_0 > 0$ with $\sigma_{\rho\theta\varphi} = +1$ for example, and this can lead to consistency problems.

One sees therefore that the sign(B_φ) depends on the cylindrical system (and thus the effective sign(B_0)), the sign(ψ_{ref}) depends on the sign(σ_{Bp}) (and on sign(I_p)), and the sign(q) on the poloidal coordinate system (and signs of I_p and B_0). We can now give the table of the relative signs and directions for the various coordinate systems. For each cylindrical coordinate orientation, one can have ψ increasing or decreasing from the magnetic axis ($\sigma_{Bp} = \pm 1$ respectively) and θ oriented counter-clockwise or clockwise, leading to q positive or negative. We have therefore 2x2x2=8 cases. In addition, the poloidal flux can be already divided by 2π or not, leading to cases 1 to 8 and 11 to 18 respectively, as detailed in Table I.

Comparing Table I and Eq. (9), we see that we have:

For COCOS = 1/11 to 4/14

$$q = \frac{1}{(2\pi)^{(1-e_{Bp})}} \frac{d\Phi_{tor}}{d\psi_{ref}}. \tag{10}$$

For COCOS = 5/15 to 8/18:

$$q = \frac{-1}{(2\pi)^{(1-e_{Bp})}} \frac{d\Phi_{tor}}{d\psi_{ref}}. \tag{11}$$

$COCOS$	e_{Bp}	σ_{Bp}	cylind,$\sigma_{R\varphi Z}$	poloid,$\sigma_{\rho\theta\varphi}$	φ from top	θ from front	ψ_{ref}	sign(q)	sign($\frac{dp}{d\psi}$)
1/11	0/1	+1	(R,φ,Z),+1	(ρ,θ,φ),+1	cnt-clockwise	clockwise	increasing	+1	-1
2/12	0/1	+1	(R,Z,φ),-1	(ρ,θ,φ),+1	clockwise	cnt-clockwise	increasing	+1	-1
3/13	0/1	-1	(R,φ,Z),+1	(ρ,φ,θ),-1	cnt-clockwise	cnt-clockwise	decreasing	-1	+1
4/14	0/1	-1	(R,Z,φ),-1	(ρ,φ,θ),-1	clockwise	clockwise	decreasing	-1	+1
5/15	0/1	+1	(R,φ,Z),+1	(ρ,φ,θ),-1	cnt-clockwise	cnt-clockwise	increasing	-1	-1
6/16	0/1	+1	(R,Z,φ),-1	(ρ,φ,θ),-1	clockwise	clockwise	increasing	-1	-1
7/17	0/1	-1	(R,φ,Z),+1	(ρ,θ,φ),+1	cnt-clockwise	clockwise	decreasing	+1	+1
8/18	0/1	-1	(R,Z,φ),-1	(ρ,θ,φ),+1	clockwise	cnt-clockwise	decreasing	+1	+1

TABLE I: Coordinate conventions for each $COCOS$ index. $COCOS \leq 8$ refers to ψ divided by (2π) and thus with $e_{Bp}=0$ while $COCOS \geq 11$ refers to full poloidal flux with $e_{Bp}=1$. Otherwise $COCOS=i$ and $COCOS=10+i$ have the same coordinate conventions. The cylindrical (with the related $\sigma_{R\varphi Z}$ value) and poloidal (with $\sigma_{\rho\theta\varphi}$) right-handed coordinate systems are given as well. The indications in the last three columns are assuming I_p and B_0 positive in the related coordinate system, that is in the direction of the related φ.

This comes from the fact that $\sigma_{Bp}\, d\mathbf{S_p}$ is in the same direction as θ near the major axis for the first four cases, hence the poloidal flux has the usual sign, while for the last four cases θ is chosen with the opposite direction. We also see from Table I, for I_p and B_0 in the same direction, q is positive when (ρ,θ,φ) is right-handed and negative otherwise.

Ultimately, one would like to have a consistent magnetic field from the equilibrium solution. The best is to check the B_R and B_Z components yielding B_p in (R,φ,Z) or (R,Z,φ) coordinates. This is how one can obtain B_R and B_Z from $\psi_{ref}(R,Z)$:

(R,φ,Z) **right-handed cylindrical coordinate system** :

$$\mathbf{B_p} = \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\nabla\varphi\times\nabla\psi_{ref} = \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\begin{pmatrix} \frac{1}{R}\,\frac{\partial\psi_{ref}}{\partial Z} \\ 0 \\ -\frac{1}{R}\,\frac{\partial\psi_{ref}}{\partial R}\end{pmatrix}, \tag{12}$$

(R, Z, φ) **right-handed cylindrical coordinate system** :

$$\mathbf{B_p} = \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\nabla\varphi \times \nabla\psi_{ref} = \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}} \begin{pmatrix} -\frac{1}{R}\frac{\partial\psi_{ref}}{\partial Z} \\ \frac{1}{R}\frac{\partial\psi_{ref}}{\partial R} \\ 0 \end{pmatrix}. \tag{13}$$

In each case, of course, one has: $B_\varphi = F/R$, that is $\mathbf{B}_\varphi = F\,\nabla\varphi$. From the above equations, one can see that if there is a plasma current in the φ direction ($I_p > 0$), then if ψ_{ref} is increasing with minor radius, $\partial\psi_{ref}/\partial R > 0$ at the LFS and B_z points downwards in the (R, φ, Z) case and upwards in the (R, Z, φ) as expected, with $\sigma_{Bp} = +1$. This is why if ψ_{ref} is decreasing with minor radius, one needs $\sigma_{Bp} = -1$ to obtain the same B_R and B_Z values. We do not discuss here the case where the φ direction is opposite in the cylindrical and the poloidal systems, since we think this case should not be used.

III. TRANSFORMATIONS OF OUTPUTS OF AN EQUILIBRIUM CODE FOR ANY COORDINATE CONVENTION

It is easier to first discuss how to transform the solution of a specific equilibrium code using a specific *COCOS* value. Let us take the case $COCOS = 2$ with the example of the CHEASE [5] code with $\psi_{ref} = \psi_{chease,2}$, defining the subscript "$_{chease,2}$" as being in CHEASE units and with the CHEASE index $COCOS = 2$. In addition, equilibrium codes usually work in normalized variables with distances normalized using a value l_d and magnetic fields using l_B as basic units. For example, CHEASE uses $l_d = R_0$ the geometrical axis and $l_B = B_0$ the vacuum field at $R = R_0$. In such a case, the equilibrium code automatically assumes I_p and B_0 positive since it works in positive normalized units.

For example, let us say we want an output following the ITER coordinate convention, $COCOS = 1$ or 11, with the flux in Webers/radian or in Webers, respectively. Taking the standard ITER case with negative I_p and B_0 in the (R, φ, Z) system, it corresponds to positive I_p and B_0 in the system with (R, Z, φ) right-handed as assumed by $COCOS = 2$ (e.g. CHEASE). First one needs to determine the relation between physical quantities in SI units (physical units) and code quantities (CHEASE or another code). They are given by

([5], p. 236 with $R_0 = l_d$ and $B_0 = l_B$):

$$
\begin{aligned}
B_{si} &= B_{chease,2}\, l_B, \\
R_{si} &= R_{chease,2}\, l_d, \\
Z_{si} &= Z_{chease,2}\, l_d, \\
\psi_{si} &= \psi_{chease,2}\, l_d^2\, l_B, \\
p_{si} &= p_{chease,2}\, l_B^2/\mu_0, \\
F_{si} &= F_{chease,2}\, l_d\, l_B, \\
\left.\frac{dp}{d\psi}\right|_{si} &= \left.\frac{dp}{d\psi}\right|_{chease,2} l_B/(\mu_0\, l_d^2), \\
F_{si}\left.\frac{dF}{d\psi}\right|_{si} &= F_{chease,2}\left.\frac{dF}{d\psi}\right|_{chease,2} l_B, \\
I_{si} &= I_{chease,2}\, l_d\, l_B/\mu_0, \\
j_{si} &= j_{chease,2}\, l_B/(\mu_0\, l_d),
\end{aligned}
\tag{14}
$$

We can now define the various transformation to the values in the new coordinate system, defining the subscript "$_{si,cocos}$" as being in SI units and with the assumptions given in Table I for the given $COCOS$ index:

$$
\begin{aligned}
\sigma_{Ip} &= sign(I_p), \\
\sigma_{B_0} &= sign(B_0), \\
\psi_{si,cocos} &= \sigma_{Ip}\,\sigma_{Bp}\,(2\pi)^{e_{Bp}}\,\psi_{chease,2}\, l_d^2\, l_B, \\
\Phi_{si,cocos} &= \sigma_{B_0}\,\Phi_{chease,2}\, l_d^2\, l_B, \\
\left.\frac{dp}{d\psi}\right|_{si,cocos} &= \frac{\sigma_{Ip}\,\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\left.\frac{dp}{d\psi}\right|_{chease,2} l_B/(\mu_0\, l_d^2), \\
F_{si,cocos}\left.\frac{dF}{d\psi}\right|_{si,cocos} &= \frac{\sigma_{Ip}\,\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\, F_{chease,2}\left.\frac{dF}{d\psi}\right|_{chease,2} l_B, \\
B_{si,cocos} &= \sigma_{B_0}\, B_{chease,2}\, l_B, \\
F_{si,cocos} &= \sigma_{B_0}\, F_{chease,2}\, l_d\, l_B, \\
I_{si,cocos} &= \sigma_{Ip}\, I_{chease,2}\, l_d\, l_B/\mu_0, \\
j_{si,cocos} &= \sigma_{Ip}\, j_{chease,2}\, l_B/(\mu_0\, l_d), \\
q_{cocos} &= \sigma_{Ip}\,\sigma_{B_0}\,\sigma_{\rho\theta\varphi}\, q_{chease,2},
\end{aligned}
\tag{15}
$$

with $\mu_0 = 4\,\pi\,10^{-7}$. $R_{si,cocos}, Z_{si,cocos}$ and $p_{si,cocos}$ are the same as R_{si}, Z_{si} and p_{si} given in Eq. (14), since they do not depend on the $COCOS$ index. Other quantities can easily be transformed using Eq. (15) and the following method, for example for $dV/d\psi$:

$$\left.\frac{dV}{d\psi}\right|_{si,cocos} = \frac{dV_{si,cocos}}{d\psi_{si,cocos}} = \frac{dV_{chease,2}\, l_d^3}{\sigma_{Ip}\,\sigma_{Bp}\,(2\pi)^{e_{Bp}}\, d\psi_{chease,2}\, l_d^2\, l_B} = \frac{\sigma_{Ip}\,\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\left.\frac{dV}{d\psi}\right|_{chease,2} \frac{l_d}{l_B}, \quad (16)$$

Taking the case of ITER with $COCOS = 11$, $\sigma_{Ip} = -1$ and $\sigma_{B_0} = -1$ so that I_p and B_0 are physically the same as in the $COCOS = 2$ system with $I_{p,chease,2}$ and $B_{0,chease,2}$ positive, we have for $COCOS = 11$ from Table I: $\sigma_{Bp} = +1$ and $e_{Bp} = 1$. Therefore we obtain:

$$\psi_{iter,11} = -\,2\pi\, \psi_{chease,2}\, R_0^2\, B_0. \quad (17)$$

Thus $\psi_{iter,11}$ will be maximum at the magnetic axis and decreasing with minor radius. This yields for example from Eq. (12):

$$B_{Z,iter,11} = -\frac{1}{R}\,\frac{\partial \psi_{iter,11}}{\partial R}. \quad (18)$$

This gives $B_{Z,iter,11} > 0$ at the LFS, which is consistent with $I_p < 0$ in the (R, φ, Z) system. Note that within CHEASE system, one has to use Eq. (13):

$$B_{Z,chease,2} = \frac{1}{R}\,\frac{\partial \psi_{chease,2}}{\partial R}. \quad (19)$$

Since $\psi_{chease,2}$ is increasing with minor radius, it also gives $B_{Z,chease,2} > 0$ as it should.

In the coordinate systems defined by the $COCOS$ value and the related values in Table I, the magnetic field should be computed as follows:

$$\begin{aligned} B_R &= \frac{\sigma_{R\varphi Z}\,\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\frac{1}{R}\,\frac{\partial \psi_{si,cocos}}{\partial Z}, \\ B_Z &= -\,\frac{\sigma_{R\varphi Z}\,\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\frac{1}{R}\,\frac{\partial \psi_{si,cocos}}{\partial R}, \\ B_\varphi &= \frac{F_{si,cocos}}{R}, \end{aligned} \quad (20)$$

where the signs of B_R and B_Z depend whether (R, φ, Z) is right-handed or not. These can be used to check the output $\psi_{si,cocos}$ and $F_{si,cocos}$ are as expected.

IV. TRANSFORMATIONS OF INPUTS FROM ANY COORDINATE CONVENTION

We can now use the inverse transformation of Eq. (15) to determine the correct inputs within a given code coordinate system (CHEASE in our example) for any assumed input coordinate system *cocos_in*. Given a coordinate *cocos_in* as defined in Table I and assuming values are in SI units, we have, given l_d, l_B, $B_{chease,2} = B_{si}/l_B$, $R_{chease,2} = R_{si}/l_d$ and $Z_{chease,2} = Z_{si}/l_d$ (for CHEASE $l_d = R_0$ and $l_B = B_0$) and imposing $\psi_{chease,2}(edge) = 0$:

$$\begin{aligned}
\psi_{chease,2} &= (\psi_{si,cocos} - \psi_{si,cocos}(edge))\, \frac{\sigma_{Ip}\,\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\, \frac{1}{l_d^2\, l_B}, \\
\left.\frac{dp}{d\psi}\right|_{chease,2} &= \left.\frac{dp}{d\psi}\right|_{si,cocos} \sigma_{Ip}\,\sigma_{Bp}\,(2\pi)^{e_{Bp}}\, \frac{\mu_0\, l_d^2}{l_B}, \\
F_{chease,2} \left.\frac{dF}{d\psi}\right|_{chease,2} &= F_{si,cocos} \left.\frac{dF}{d\psi}\right|_{si,cocos} \sigma_{Ip}\,\sigma_{Bp}\,(2\pi)^{e_{Bp}}\, \frac{1}{l_B}, \\
I_{p,chease,2} &= I_{p,si,cocos}\; \sigma_{Ip}\, \frac{\mu_0}{l_d\, l_B}, \\
j_{chease,2} &= j_{si,cocos}\; \sigma_{Ip}\, \frac{\mu_0\, l_d}{l_B}, \\
q_{chease,2} &= q_{cocos}\; \sigma_{Ip}\,\sigma_{B_0}\,\sigma_{\rho\theta\varphi},
\end{aligned} \tag{21}$$

and the plasma boundary is normalized by l_d. Since CHEASE assumes (R, Z, φ) and (ρ, θ, φ) right-handed and I_p, B_0 positive, we check the input consistency with:

$$\begin{aligned}
\psi_{chease,2} &: \text{ should be minimum at magnetic axis,} \\
I_{p,chease,2} &: \text{ should be positive,} \\
\left.\frac{dp}{d\psi}\right|_{chease,2} &: \text{ should be negative,} \\
q_{chease,2} &: \text{ should be positive.}
\end{aligned} \tag{22}$$

A similar check of the final input values will apply to any code other than CHEASE. Note that the sign of q may not be consistent with the other quantities since it is often given as $abs(q)$. Therefore a warning should be issued if q is not consistent but the input should not be rejected.

If the input is an eqdsk file as described in [5] (p. 236), then we also have:

$$\begin{aligned}
F_{chease,2} &= F_{si,cocos}\, \frac{\sigma_{B_0}}{l_d\, l_B}, \\
p_{chease,2} &= p_{si,cocos} \frac{\mu_0}{l_B^2},
\end{aligned}$$

with the check that $F_{chease,2}$ should be positive and $F_{chease,2}$(edge)$= +1$ (since in this example $F_{si}(edge) = R_0 B_0 = l_d l_B$). The value of $p_{chease,2}$(edge) is typically used to impose the edge pressure.

V. CHECKING THE CONSISTENCY OF EQUILIBRIUM QUANTITIES/ASSUMPTION WITH A *COCOS* INDEX

Let us obtain conditions of consistency of an input equilibrium with a specific $COCOS$ index, generalizing Eq. (22). For this, it is easier to use Eq. (21) and to note that since with the CHEASE normalization we have I_p and B_0 positive, we should have I_p and F positive, ψ_{chease} increasing, $dp/d\psi_{chease}$ negative and q positive (from Table I, $COCOS = 2$ line). Thus, using Eq. (21) we should have for any $cocos$ equilibrium:

$$
\begin{aligned}
\sigma_{Ip} &= sign(I_p), \\
\sigma_{B_0} &= sign(B_0), \\
sign(F_{cocos}) &= \sigma_{B_0}, \\
sign(\Phi_{cocos}) &= \sigma_{B_0}, \\
sign\big[\psi_{cocos}(\text{edge}) - \psi_{\text{cocos}}(\text{axis})\big] &= \sigma_{Ip}\ \sigma_{Bp,cocos}, \\
sign(\frac{dp}{d\psi}\Big|_{cocos}) &= -\ \sigma_{Ip}\ \sigma_{Bp,cocos}, \\
sign(j_{cocos}) &= \sigma_{Ip}, \\
sign(q_{cocos}) &= \sigma_{Ip}\ \sigma_{B_0}\ \sigma_{\rho\theta\varphi},
\end{aligned}
\tag{23}
$$

with $\sigma_{Bp,cocos}, \sigma_{\rho\theta\varphi}$ given in Table I for the related $cocos$ value. Note that the sign of $dp/d\psi$ being $-\sigma_{Ip}\sigma_{Bp,cocos}$ should be understood as the "main" $sign(dp/d\psi)$ following the fact that pressure is usually much larger on axis than at the edge. To be more precise one could replace this relation by $sign\big(\sum_0^{\text{edge}} \frac{dp}{d\psi}\,\Delta\psi\big) = -1$.

It should be noted that Eq. (23) can also be used to determine the $COCOS$ used in a code or set of equations. Usually, one starts by checking if ψ is increasing or decreasing from magnetic axis to the edge. Then, depending on $sign(I_P)$, one can obtain the value of $\sigma_{Bp,cocos}$. Another way is if $\mathbf{B}_p \sim \nabla\varphi \times \nabla\psi$, thus $\sigma_{Bp,cocos} = +1$ or $\mathbf{B}_p \sim \nabla\psi \times \nabla\varphi$, yielding $\sigma_{Bp,cocos} = -1$. Then one can check with the sign of $dp/d\psi$. The next step is to determine $\sigma_{R\varphi Z}$, either from the comparison of the sign of I_p and B_0 with the effective direction of I_p and B_0 if it is known, or by comparing the definition of B_R, for example, with Eqs. (12) and (13) and taking into account the value of σ_{Bp}. Then, the effective sign of q gives the value of $\sigma_{\rho\theta\varphi}$. Finally, e_{Bp} is obtained from the factor 2π appearing either in the definition of $\mathbf{B}_p$, Eq. (1), giving $e_{Bp} = 1$ or in the definition of q, Eq. (9), yielding $e_{Bp} = 0$. Note that if a specific sign of I_p or B_0 is used, it should be used in Eq. (23) to infer the $COCOS$ value. In particular, some codes (Table IV) use a different sign for I_p and B_0, yielding a different effective sign of q.

VI. TRANSFORMATIONS FROM ANY INPUT *cocos_in* TO ANY OUTPUT *cocos_out*

The above transformations, Eqs. (15) and (21), are generic however represent the transformation from $COCOS = 2$ to any $cocos\_out$ and from any $cocos\_in$ to $COCOS = 2$, respectively. The easiest way to obtain the direct general transformation is to combine the two transformations sequentially, replacing "$si, cocos$" in Eq. (15) by "$si, cocos\_out$" and the related parameters σ_{Bp}, e_{Bp}, l_d etc by $\sigma_{Bp,cocos\_out}$, $e_{Bp,cocos\_out}$, $l_{d,out}$, etc. Similarly, in Eq. (21) one changes "$si, cocos$" with "$si, cocos\_in$" and σ_{Bp}, e_{Bp}, l_d, etc with $\sigma_{Bp,cocos\_in}$, $e_{Bp,cocos\_in}$, $l_{d,in}$, etc. We can then eliminate $\psi_{chease,2}$, $I_{p,chease,2}$, etc to obtain the transformation from an "input" equilibrium with $COCOS = cocos\_in$ to an "output" equilibrium with $COCOS = cocos\_out$, taking also into account any differences in normalization. At the end, for the coordinate transformations, it gives similar equations to Eq. (15) with "$cocos$" replaced by "$cocos\_out$", "$chease, 2$" by "$si, cocos\_in$" and with:

$$\begin{aligned}
\sigma_{Bp} &\rightarrow \sigma_{Bp,cocos_out}\ \sigma_{Bp,cocos_in}, \\
e_{Bp} &\rightarrow e_{Bp,cocos_out} - e_{Bp,cocos_in}, \\
\sigma_{\rho\theta\varphi} &\rightarrow \sigma_{\rho\theta\varphi,cocos_out}\ \sigma_{\rho\theta\varphi,cocos_in}, \\
\sigma_{Ip} &\rightarrow \sigma_{Ip,cocos_out}\ \sigma_{Ip,cocos_in}, \\
\sigma_{B0} &\rightarrow \sigma_{B0,cocos_out}\ \sigma_{B0,cocos_in}.
\end{aligned} \tag{24}$$

Note that the sign of I_p for example should be transformed according to the relative directions of φ in the two coordinate systems, therefore depending on the sign of $(\sigma_{R\varphi Z,cocos\_out}\ \sigma_{R\varphi Z,cocos\_in})$. The values of the parameters for the various $COCOS$ systems are all given in Table I. In order to be more precise, we provide the explicit relations in Appendix C for both the signs transformations and for the normalizations. In addition we discuss the case of a mere transformation of coordinates and the case when a given sign of I_p and/or B_0 are required.

VII. SIMILARITY BETWEEN $COCOS = 1$ AND $COCOS = 2$ AND EFFECTS OF CHANGING SIGNS OF I_p AND B_0

Looking at Table I, one sees that $COCOS = 1$ and $COCOS = 2$ give the same values of e_{Bp} and σ_{Bp} and all the other parameters listed (similar remarks apply to $COCOS$ pairs 11 and 12, 3 and 7, etc). This is the case for CHEASE-like and ITER-like assumptions. But what does it mean and where is the difference between the two systems?

First, it means that they have the same $\mathbf{B}$ representation in terms of the same Eq. (1), since it depends only on e_{Bp} and σ_{Bp}. However, in this case the respective φ are in opposite direction. Therefore for a given real case, say a standard ITER case with I_p and B_0 clockwise, then σ_{Ip} and σ_{B_0} will be opposite (namely -1 for $COCOS = 1$ and +1 for $COCOS = 2$). In addition, the equations to evaluate B_R and B_Z are different since $COCOS = 1$ should use Eq. (20) with $\sigma_{R\varphi Z} = +1$, while $COCOS = 2$ has $\sigma_{R\varphi Z} = -1$. This is how the final B_R and B_Z are the same at the end, since they should not depend on the coordinate system, as discussed near Eqs. (18-19). For the cases $COCOS = 3$ and $COCOS = 7$, there is no difference in the cylindrical coordinate systems, therefore R, Z projections are the same. The only difference is with respect to θ such that B_p is along θ with $COCOS = 7$ (hence q is positive) and opposite to θ with $COCOS = 3$ (hence q is negative) with I_p and B_0 positive. In this case, only the sign of q differentiates the two systems. On the other hand, one often provides only $abs(q)$ in the output (since $q < 0$ is unusual) and therefore it might be difficult to know the effective assumed coordinate convention and the sign of the poloidal flux (hence the usefulness of specifying the $COCOS$ value). One way is to check the variations with the signs of I_p and B_0. In Table I, only the case with I_p and B_0 positive are given. However it is good to check the variations when changing both σ_{Ip} and σ_{B_0}. For example, for the first case, $COCOS = 1$ or 11, Table II shows the effects of varying the experimental sign of I_p and/or B_0. For the general case, $COCOS = cosos$, the relative signs of the main equilibrium quantities are provided in Table III in terms of the effective signs of I_p and/or B_0.

$COCOS$	σ_{Ip}	σ_{B_0}	ψ_{ref}	$sign(q)$	$sign(dp/d\psi)$	$sign(F)$	$sign(FdF/d\psi)$
1/11	+1	+1	increasing	positive	negative	positive	ii
1/11	-1	+1	decreasing	negative	positive	positive	$-ii$
1/11	+1	-1	increasing	negative	negative	negative	$+ii$
1/11	-1	-1	decreasing	positive	positive	negative	$-ii$

TABLE II: Signs of related quantities when I_p or B_0 change sign in the case of $COCOS = 1$ or 11. In the last column, the sign relative to the first case ($ii = \pm 1$) is given. For other coordinate systems, the effect of changing σ_{Ip} on ψ_{ref}, q, $dp/d\psi$ and $FdF/d\psi$ is similar, as well as changing σ_{B_0} on q and $sign(F)$. This can be deduced from Table I for the first line (with $\sigma_{Ip} = \sigma_{B_0} = +1$) and Eq. (15) for the other signs of I_p and B_0.

$COCOS$	σ_{Ip}	σ_{B_0}	$d\psi_{ref}$	$sign(q)$	$sign(dp/d\psi)$	$sign(F)$	$sign(FdF/d\psi)$
$cocos$	+1	+1	σ_{Bp}	$\sigma_{\rho\theta\varphi}$	$-\sigma_{Bp}$	+1	$-\sigma_{Bp}\, ii$
$cocos$	-1	+1	$-\sigma_{Bp}$	$-\,\sigma_{\rho\theta\varphi}$	σ_{Bp}	+1	$\sigma_{Bp}\, ii$
$cocos$	+1	-1	σ_{Bp}	$-\,\sigma_{\rho\theta\varphi}$	$-\sigma_{Bp}$	-1	$-\sigma_{Bp}\, ii$
$cocos$	-1	-1	$-\sigma_{Bp}$	$\sigma_{\rho\theta\varphi}$	σ_{Bp}	-1	$\sigma_{Bp}\, ii$

TABLE III: Signs of related quantities when I_p or B_0 change sign in the general case of $COCOS = cocos$. The respective values of σ_{Bp} and $\sigma_{\rho\theta\varphi}$ are given in Table I.

VIII. THE GRAD-SHAFRANOV EQUATION

It is also useful to rewrite the Grad-Shafranov equation in terms of the generic $\mathbf{B}$ of Eq. (1), since it allows to transform the source terms correctly when changing to a new $\psi_{si,cocos}$ definition. First we need to calculate $\mathbf{j}$ assuming $F = F(\psi_{ref})$ and $p = p(\psi_{ref})$ and (R, φ, Z) right-handed ($\sigma_{R\varphi Z} = +1$ in Eq. (20)):

$$\mathbf{j} = \frac{1}{\mu_0}\,\nabla \times \mathbf{B} = \frac{1}{\mu_0}\begin{pmatrix} -\frac{1}{R}\frac{dF}{d\psi_{ref}}\frac{\partial\psi_{ref}}{\partial Z} \\ \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\frac{1}{R}\left[\frac{\partial^2\psi_{ref}}{\partial Z^2} + R\,\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\psi_{ref}}{\partial R}\right] \\ \frac{1}{R}\frac{dF}{d\psi_{ref}}\frac{\partial\psi_{ref}}{\partial R} \end{pmatrix}, \tag{25}$$

which can be re-written, with $F' = dF/d\psi_{ref}$ and $\Delta^* \equiv \frac{\partial^2\psi_{ref}}{\partial Z^2} \ + R\,\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\psi_{ref}}{\partial R}$:

$$\mathbf{j} = \frac{1}{\mu_0}\,\Big(-\frac{(2\pi)^{e_{Bp}}}{\sigma_{Bp}}F'\,\mathbf{B} + \big[\frac{(2\pi)^{e_{Bp}}}{\sigma_{Bp}}\ FF' + \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\Delta^*\psi_{ref}\big]\nabla\varphi\Big). \tag{26}$$

In the above, we have used the (R, φ, Z) cylindrical system. If we have the (R, Z, φ) system with $\sigma_{R\varphi Z} = -1$ in Eq. (20), we get:

$$\mathbf{j} = \frac{1}{\mu_0}\,\nabla\times\mathbf{B} = \frac{1}{\mu_0}\,\begin{pmatrix} \frac{1}{R}\,F'\,\frac{\partial\psi_{ref}}{\partial Z} \\ -\,\frac{1}{R}\,F'\,\frac{\partial\psi_{ref}}{\partial R} \\ \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\,\frac{1}{R}\,\big[\frac{\partial^2\psi_{ref}}{\partial Z^2}\ +\ R\,\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\psi_{ref}}{\partial R}\big] \end{pmatrix}, \tag{27}$$

which also gives:

$$\mathbf{j} = \frac{1}{\mu_0}\,\Big(-\frac{(2\pi)^{e_{Bp}}}{\sigma_{Bp}}F'\mathbf{B} + \big[\frac{(2\pi)^{e_{Bp}}}{\sigma_{Bp}}\ FF' + \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\Delta^*\psi_{ref}\big]\nabla\varphi\Big). \tag{28}$$

We now use the static equilibrium equation, $\nabla p = \mathbf{j}\times\mathbf{B}$, and introduce Eqs. (26, 28) and (1):

$$p'\,\nabla\psi_{ref} = \frac{1}{\mu_0}\big[\frac{(2\pi)^{e_{Bp}}}{\sigma_{Bp}}\ FF' + \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}\Delta^*\psi_{ref}\big]\nabla\varphi\ \cdot\ \frac{\sigma_{Bp}}{(2\pi)^{e_{Bp}}}(\nabla\varphi\times\nabla\psi_{ref}), \tag{29}$$

which yields, using $\sigma_{Bp}^2 = 1$:

$$\Delta^*\psi_{ref} = -\ \mu_0\ (2\pi)^{2e_{Bp}}\ R^2\ p' -\ (2\pi)^{2e_{Bp}}\ FF' = \sigma_{Bp}\ (2\pi)^{e_{Bp}}\ \mu_0\ R\ j_\varphi. \tag{30}$$

We see that indeed it does not depend anymore on σ_{Bp}, nor on $\sigma_{\rho\theta\varphi}$ nor if it is (R, φ, Z) or (R, Z, φ) which is right-handed. Taking $\psi_{ref} = \psi_{1-8}$ with $e_{Bp} = 0$, that is $COCOS = 1$ to 8, we have the usual Grad-Shafranov equation:

$$\Delta^*\psi_{1-8} = -\ \mu_0\ R^2\ p' -\ FF' = \sigma_{Bp}\ \mu_0\ R\ j_\varphi, \tag{31}$$

with $p' = dp/d\psi_{1-8}$ and similarly for F'. If we would now use $\psi_{11-18} = 2\pi\,\psi_{1-8}$, we have $dp/d\psi_{1-8} = 2\pi\,dp/d\psi_{11-18}$ and introducing it with $\psi_{1-8} = \psi_{11-18}/2\pi$ we get:

$$\Delta^*\psi_{11-18} = -\ \mu_0\ R^2\ (2\pi)^2\,p' -\ (2\pi)^2\,FF' = \sigma_{Bp}\,2\,\pi\ \mu_0\ R\ j_\varphi, \tag{32}$$

with this time $p' = dp/d\psi_{11-18}$ and similarly for F' and we recover Eq. (30) with $e_{Bp} = 1$. Thus it does not depend on e_{Bp} either except that ψ might be rescaled as well as the source functions p' and FF'.

IX. CONCLUSIONS

We have defined a new single parameter $COCOS$ to determine the coordinate systems used for the cylindrical and poloidal coordinates, the sign of the poloidal flux and whether ψ is divided by (2π) or not. This is defined in Table I. This allows a generic definition of the magnetic field $\mathbf{B}$ (Eq. (1)) using only two new parameters: σ_{Bp} and e_{Bp}. These parameters are also defined uniquely by the parameter $COCOS$ in Table I. This $COCOS$ parameter is useful to define the assumptions used by a specific code. All the various options are contained within the 16 cases defined in Table I. It should be emphasized that there are "only" 16 cases because we have assumed Z upwards in the cylindrical system and φ in the same direction for the cylindrical and poloidal coordinate systems.

We have also defined the procedure to transform input values from any of the 16 cases of Table I to a given code assumptions, in our example for $COCOS = 2$ case, in order to provide the code with self-consistent input values (Sec. IV, Eq. (21)). Similarly, Sec. III defines the transformations required for a code with $COCOS = 2$ to provide output values consistent with any of the 16 cases defined in Table I. In this case, not only the value of *cocos_out* needs to be specified, but also l_d, l_B, σ_{Ip} and σ_{B_0} as given by Eq. (15). This allowed us to define consistency checks of an equilibrium with a given *cocos* value and to propose a procedure to determine the *cocos* index assumed in a code or a set of equations (Sec. V).

The general equilibrium transformations from any *cocos_in* to any *cocos_out* convention is given in Sec. VI and Appendix C, including the transformation of the normalizations and how to simply change the sign of I_p and/or B_0.

The correct definitions of B_R, B_Z and B_φ are given in Eq. (20), of q with respect to toroidal flux in Eqs. (10) and (11) and of the sources and the Grad-Shafranov equation in Eqs. (31) and (32). The effect of changing the signs of I_p and/or B_0 are provided in Table II for ITER ($COCOS = 11$) and in Table III for the general case.

As mentioned above, Table I can be used to define the coordinate conventions of a given code or set of equations. For example, CHEASE [5] and ONETWO [7] use $COCOS = 2$, ITER [10] should use $COCOS = 11$, the EU-ITM [8] was using $COCOS = 13$ up to the end of 2011 and the TCV tokamak is using $COCOS = 7$ and 17 [11], [12]. The code ORB5 [13] uses $COCOS = 3$ but, to have q positive, normalizes the plasma current such that it is negative. This is another way to resolve the "problem" mentioned in the Introduction. The Table in Appendix A shall keep track of the known *COCOS* choices and the various ways the authors of these codes have resolved the relation between cylindrical and poloidal coordinate systems. On the other hand, the choice of the poloidal angle "θ" is not discussed here, for example if straight-field line coordinates are assumed. We shall also keep track of the assumed coordinate conventions for the various tokamaks and other magnetic devices when relevant. These are provided in Appendix B. Note that this is also important for diagnostics which might be related to a given sign convention of the coordinate systems, like the toroidal and poloidal rotations, as discussed near Eq. (48),

The main aim of this paper is to contribute to establishing well-defined interfaces and providing useful reference information in support of the current world-wide ITER Integrating Modelling efforts.

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References

[1] V.D. Shafranov, ZhETF **33** (1957) 710; Sov. Phys. JETP **8** (1958) 494.

[2] R. Lüst and A. Schlüter, Z. Naturforsch. **129** (1957) 850.

[3] H. Grad and H. Rubin, Proc. 2nd Int. Conf. on the Peaceful Uses of Atomic Energy, Vol. **31** (United Nations, Geneva, 1958) p. 190.

[4] J. P. Freidberg, *Ideal magnetohydrodynamic theory of magnetic fusion systems*, Rev. Mod. Phys. **54** (1982) 801-902.

[5] H. Lütjens, A. Bondeson and O. Sauter, *The CHEASE code for toroidal MHD equilibria*, Computer Physics Communications **97** (1996) 219-260.

[6] F. L. Hinton and R. D. Hazeltine, *Theory of plasma transport in toroidal confinement systems*, Rev. Mod. Phys. **48** (1976) 239

[7] W.W. Pfeiffer , R.H. Davidson , R.L. Miller and R.E. Waltz, *ONETWO: A computer code for modeling plasma transport in tokamaks*, General Atomics Company Report, 1980. GA-A16178

[8] F. Imbeaux et al, *A generic data structure for integrated modelling of tokamak physics and subsystems*, Comput. Phys. Commun. **181** (2010) 987

[9] A. H. Boozer, *Physics of magnetically confined plasmas*, Rev. Mod. Phys. **76**(2005) 1071-1141

[10] *ITER Integrated Modelling Standards and Guidelines*, IDM ITER document 2F5MKL, version 2.1, Sept. 5 2010, p. 7-10

[11] J.-M. Moret, *A software package to manipulate space dependencies and geometry in magnetic confinement fusion*, Rev. Sci. Instrum. **69** (1998) 2333.

[12] F. Hofmann and G. Tonetti, *Tokamak equilibrium reconstruction using Faraday rotation measurements*, Nucl. Fusion **28** (1988) 1871.

[13] S. Jolliet, A. Bottino, P. Angelino, R. Hatzky, T. M. Tran, B. F. McMillan, O. Sauter, K. Appert, Y. Idomura, and L. Villard, Comput. Phys. Commun. **177** (2007) 409.

[14] http://crpp.epfl.ch/˜sauter/CHEASE

[15] L. Villard, K. Appert, R. Gruber, J. Vaclavik, *GLOBAL WAVES IN COLD PLASMAS*, Computer Physics Reports **4** (1986) 95

[16] H.Lütjens, J.F.Luciani, *XTOR-2F : a fully implicit Newton-Krylov solver applied to nonlinear*

3D extended MHD in tokamaks, Journal of Comp. Physics **229** (2010) 8130.

[17] T. TAKEDA and S. TOKUDA, *Computation of MHD Equilibrium of Tokamak Plasma*, Journal of Computational Physics **93** (1991) 1.

[18] T. Görler, X. Lapillonne, S. Brunner, T. Dannert, F. Jenko, F. Merz, and D. Told, *The Global Version of the Gyrokinetic Turbulence Code GENE*, Journal of Computational Physics **230** (2011) 7053.

[19] J. P. Freidberg, *Ideal magnetohydrodynamics*, Plenum Press, New York, 1987, ISBN 0-306-42512-2, p. 108-110

[20] L. Degtyarev, A. Martynov, S. Medvedev, F. Troyon, L. Villard and R. Gruber, *The KINX ideal MHD stability code for axisymmetric plasmas with separatrix*, Computer Physics Communications **103** (1997) 10.

[21] D. Farina, *A Quasi-Optical Beam-Tracing Code for Electron Cyclotron Absorption and Current Drive: GRAY*, Fusion Science and Technology **52** (2007) 154.

[22] P. Ricci and B. N. Rogers, Phys. Rev. Lett. **104** (2010) 145001.

[23] Y. Idomura, M. Ida, T. Kano, N. Aiba, *Conservative global gyrokinetic toroidal full-f five-dimensional Vlasov simulation*, Comp. Phys. Comm. **179** (2008) 391

[24] E. Poli, A.G. Peeters, G.V. Pereverzev, *TORBEAM, a beam tracing code for electron-cyclotron waves in tokamak plasmas*, Comp. Phys. Comm. **136** (2001) 90

[25] A. Bondeson, G. Vlad, and H. Ltjens. *Computation of resistive instabilities in toroidal plasmas.* In IAEA Technical Committee Meeting on Advances in Simulations and Modelling of Thermonuclear Plasmas, Montreal, 1992, page 306, Vienna, Austria, 1993. International Atomic Energy Agency

Appendix A: Known $COCOS$ values for codes and set of equations

The Table below shall keep track of the known $COCOS$ values and an up-to-date version shall be maintained on the CHEASE website [14]. This applies to axisymmetric cases, but can be useful for 3D as well.

$COCOS$	codes, papers, books, etc
1	psitbx(various options) [11]
11	ITER [10], Boozer[9]
2	CHEASE [5], ONETWO [7], Hinton-Hazeltine [6], LION [15], XTOR [16], MEUDAS [17], MARS [25]
12	GENE [18]
3	Freidberg* [4], [19], CAXE and KINX* [20], GRAY [21], with $\sigma_{Ip} = -1, \sigma_{B0} = +1$: ORB5 [13], GBS [22] with $\sigma_{Ip} = -1, \sigma_{B0} = -1$: GT5D [23]
13	EU-ITM up to end of 2011 [8]
4	
14	
5	TORBEAM [24]
15	
6	
16	
7	
17	LIUQE* [12], psitbx(TCV standard output) [11]
8	
18	

TABLE IV: For each coordinate conventions index $COCOS$, this table lists known codes, papers, books that explicitely use it. The * marks that in these cases $abs(q)$ is effectively used (since ideal axisymmetric MHD does not depend on its sign). Most codes use normalized units and therefore use typically I_p and B_0 positive, as discussed in the paper for CHEASE [5]. This is not mentioned in this table. Some codes normalize such that I_p and/or B_0 is negative. This is marked explicitely in this table. This table shall be maintained and available at [14]. Send an email for a new entry.

Appendix B: Known Tokamak coordinate conventions and relation to *COCOS* values

The Table below shall keep track of the known coordinate conventions assumed by the various tokamaks. This means in particular the direction of a positive toroidal current, magnetic field and poloidal current in the coils for example. This should also help to check if, for example, the direction of positive toroidal and poloidal velocities are in the same direction as positive toroidal and poloidal currents. We also give the *COCOS* values which are compatible with the related assumptions. Since there is 2 choices for the cylindrical coordinate convention and 2 for the poloidal direction, there are 4 different cases and thus 4 *COCOS* values compatible for each case. An up-to-date version of this table shall be maintained on the CHEASE website [14].

cylind,$\sigma_{R\varphi Z}$	poloid,$\sigma_{\rho\theta\varphi}$	φ from top	θ from front	*COCOS*	Tokamaks
(R,φ,Z),+1	(ρ,θ,φ),+1	cnt-clockwise	clockwise	1/11, 7/17	TCV-magnetics, ITER [10]
(R,φ,Z),+1	(ρ,φ,θ),-1	cnt-clockwise	cnt-clockwise	3/13, 5/15	
(R,Z,φ),-1	(ρ,θ,φ),+1	clockwise	cnt-clockwise	2/12, 8/18	
(R,Z,φ),-1	(ρ,φ,θ),-1	clockwise	clockwise	4/14, 6/16	

TABLE V: Known Tokamak coordinate conventions and relation to *COCOS* values.

Appendix C: Equilibrium transformations: new *COCOS*, new I_p or B_0 sign, new normalization

There are three kinds of transformation that one might want to apply to a given equilibrium. First, of course, the transformation of an equilibrium obtained within a given $COCOS = cocos\_in$ convention into an equilibrium consistent with a new $COCOS = cocos\_out$ equilibrium. Since the solution of the Grad-Shafranov equation is independent of the $COCOS$ value, as seen in Sec. VIII, one can easily transform from one to another. Two examples have been given to and from any $COCOS$ from and to $COCOS = 2$, respectively, in Eqs. (15) and (21). The second typical transformation is to obtain a specific sign of I_p and/or B_0. Finally, one might want to normalize in one way or another as also discussed near Eqs. (15) and (21).

Let us first describe the detailed transformations from *cocos_in* to *cocos_out*. Following Sec. VI, we use Eq. (15) for the *cocos_out* values and Eq. (21) for the *cocos_in* cases and we can rewrite the first relation in each as follows:

$$\begin{aligned}
\psi_{si,cocos_out} &= \sigma_{Ip,out}\,\sigma_{Bp,cocos_out}\,(2\pi)^{e_{Bp,cocos_out}}\,\psi_{chease,2}\,l_{d,out}^2\,l_{B,out},\\
\psi_{chease,2} &= \frac{\sigma_{Ip,in}\,\sigma_{Bp,cocos_in}}{(2\pi)^{e_{Bp,cocos_in}}}\,\psi_{si,cocos_in}\,\frac{1}{l_{d,in}^2\,l_{B,in}}.
\end{aligned} \tag{33}$$

Eliminating $\psi_{chease,2}$ we obtain:

$$\psi_{si,cocos\_out} = \left(\sigma_{Ip,out}\sigma_{Ip,in}\right)\left(\sigma_{Bp,cocos\_out}\sigma_{Bp,cocos\_in}\right)(2\pi)^{[e_{Bp,cocos\_out}-e_{Bp,cocos\_in}]} \tag{34}$$

$$\psi_{si,cocos\_in}\,\frac{l_{d,out}^2\,l_{B,out}}{l_{d,in}^2\,l_{B,in}}, \tag{35}$$

which can be rewritten in a generic form exactly similar to Eq. (15):

$$\psi_{si,cocos\_out} = \tilde{\sigma}_{Ip}\,\tilde{\sigma}_{Bp}\,(2\pi)^{\tilde{e}_{Bp}}\,\psi_{si,cocos\_in}\,\tilde{l}_d^2\,\tilde{l}_B. \tag{36}$$

Thus we only need to define the $\tilde{}$ parameters to be used in Eq. (15). Comparing Eqs. (34) and (36), we have already the main parameters. Similarly we should have:

$$\tilde{\sigma}_{R\varphi Z} = \sigma_{R\varphi Z,cocos\_out}\,\sigma_{R\varphi Z,cocos\_in}, \tag{37}$$

$$\tilde{\sigma}_{\rho\theta\varphi} = \sigma_{\rho\theta\varphi,cocos\_out}\,\sigma_{\rho\theta\varphi,cocos\_in}. \tag{38}$$

The parameter in Eq. (37) does not appear explicitly in the transformations, Eq. (15), however it relates directly the effective sign of I_p or B_0 in one system to the other. Indeed, if the φ directions in the two systems are opposite, then the effective sign should change. We can see that in Eq. (34) we have labelled the I_p sign as $\sigma_{Ip,out}$ instead of $\sigma_{Ip,cocos\_out}$. This is done on purpose to emphasize the fact that the I_p sign is not necessarily related to the coordinate convention, but could be just requested in output. For example, some codes request a specific sign of I_p and B_0, being positive or negative, as seen in table IV. Therefore we have:

$$\sigma_{Ip,out} = \begin{cases} \sigma_{Ip,in}\,\tilde{\sigma}_{R\varphi Z} & \text{if a specific } \sigma_{Ip,out} \text{ is not requested} \\ \sigma_{Ip,out} & \text{otherwise} \end{cases} \tag{39}$$

Including this into $\tilde{\sigma}_{Ip} = \sigma_{Ip,out}\sigma_{Ip,in}$ and using Eq. (37), we can define directly:

$$\tilde{\sigma}_{Ip} = \begin{cases} \sigma_{R\varphi Z,cocos\_out}\,\sigma_{R\varphi Z,cocos\_in} & \text{if a specific } \sigma_{Ip,out} \text{ is not requested} \\ \sigma_{Ip,in}\,\sigma_{Ip,out} & \text{otherwise} \end{cases} \tag{40}$$

Similarly we have:

$$\tilde{\sigma}_{B0} = \begin{cases} \sigma_{R\varphi Z,cocos\_out}\,\sigma_{R\varphi Z,cocos\_in} & \text{if a specific } \sigma_{B0,out} \text{ is not requested} \\ \sigma_{B0,in}\,\sigma_{B0,out} & \text{otherwise} \end{cases} \tag{41}$$

And the other parameters are defined by:

$$\begin{aligned} \tilde{\sigma}_{Bp} &= \sigma_{Bp,cocos\_out}\sigma_{Bp,cocos\_in} \\ \tilde{e}_{Bp} &= e_{Bp,cocos\_out} - e_{Bp,cocos\_in} \end{aligned} \tag{42}$$

$$\tilde{\sigma}_{\rho\theta\varphi} = \sigma_{\rho\theta\varphi,cocos\_out}\,\sigma_{\rho\theta\varphi,cocos\_in}. \tag{43}$$

For the normalizations, it is a bit more complicated since we have the term μ_o which disappears with normalized units. The best way is to compare with the Grad-Shafranov equation written in a generic form, inspired by Eq. (30), or with $<\mu_0\, j_\varphi/R>$, related to $dI_p(\psi)/d\psi$ with $I_(\psi)$ the toroidal current within the ψ flux surface, since one of these two equations is usually well defined within a given code related to equilibrium quantities:

$$\Delta^*\psi \;=\; -\,(2\pi)^{2e_{Bp}}\,R^2\,\mu_0^{e_{\mu0}}\,p' - \;(2\pi)^{2e_{Bp}}\,FF' \tag{44}$$

$$<\mu_o\, j_\varphi/R> \;=\; -\,\sigma_{Bp}\,(2\pi)^{e_{Bp}}\,\left(\mu_0^{e_{\mu0}}\,p' \,+\, FF'\; <1/R^2>\right). \tag{45}$$

Typically, one has $e_{\mu0} = 0$ for codes using normalized units and $e_{\mu0} = 1$. To check the dimensions, one can note that from $\mathbf{B}_p \sim \nabla\varphi \times \nabla\psi$, the natural dimension of $[\psi]$ is $[l_d^2\, l_B]$

and from Maxwell's equation $\nabla \times \mathbf{B} \sim \mu_o\, j$, we have $[\mu_o\, j] \sim [l_B/l_d]$. It follows that the "natural" dimensions for the source terms are $[\mu_o p'] = [l_B/l_d^2]$ and $[FF'] = [l_B]$.

We can now define $l_{d,in}, l_{B,in}, e_{\mu0,in}$ as the characteristic length and magnetic field strength and μ_0 exponent of the input equilibrium and $l_{d,out}, l_{B,out}, e_{\mu0,out}$ of the output equilibrium ($l_{d,chease} = 1, l_{B,chease} = 1, e_{\mu0,chease} = 0$ in the case of CHEASE and $l_{d,si} = R_0, l_{B,si} = B_0, e_{\mu0,s1} = 1$ in the case of standard SI units with $F(\text{edge}) = \mathrm{R_0 B_0}$) and the corresponding tilde values:

$$
\begin{aligned}
\tilde{l}_d &= \frac{l_{d,out}}{l_{d,in}} \\
\tilde{l}_B &= \frac{l_{B,out}}{l_{B,in}}, \\
\tilde{e}_{\mu0} &= e_{\mu0,out} - e_{\mu0,in},
\end{aligned}
\tag{46}
$$

Using Eqs. (40, 41, 42) and (46) we have the general transformation from an input equilibrium to an output equilibrium given by:

$$
\begin{aligned}
\psi_{cocos\_out} &= \tilde{\sigma}_{Ip}\, \tilde{\sigma}_{Bp}\, (2\pi)^{\tilde{e}_{Bp}}\, \psi_{cocos\_in}\, \tilde{l}_d^2\, \tilde{l}_B, \\
\Phi_{cocos\_out} &= \tilde{\sigma}_{B_0}\, \Phi_{cocos\_in}\, \tilde{l}_d^2\, \tilde{l}_B, \\
\left.\frac{dp}{d\psi}\right|_{cocos\_out} &= \frac{\tilde{\sigma}_{Ip}\, \tilde{\sigma}_{Bp}}{(2\pi)^{\tilde{e}_{Bp}}} \left.\frac{dp}{d\psi}\right|_{cocos\_in} \tilde{l}_B/(\mu_0^{\tilde{e}_{\mu0}}\, \tilde{l}_d^2), \\
F_{cocos\_out} \left.\frac{dF}{d\psi}\right|_{cocos\_out} &= \frac{\tilde{\sigma}_{Ip}\, \tilde{\sigma}_{Bp}}{(2\pi)^{\tilde{e}_{Bp}}}\, F_{cocos\_in} \left.\frac{dF}{d\psi}\right|_{cocos\_in} \tilde{l}_B, \\
B_{cocos\_out} &= \tilde{\sigma}_{B_0}\, B_{cocos\_in}\, \tilde{l}_B, \\
F_{cocos\_out} &= \tilde{\sigma}_{B_0}\, F_{cocos\_in}\, \tilde{l}_d\, \tilde{l}_B, \\
I_{cocos\_out} &= \tilde{\sigma}_{Ip}\, I_{cocos\_in}\, \tilde{l}_d\, \tilde{l}_B/\mu_0^{\tilde{e}_{\mu0}}, \\
j_{cocos\_out} &= \tilde{\sigma}_{Ip}\, j_{cocos\_in}\, \tilde{l}_B/(\mu_0^{\tilde{e}_{\mu0}}\, \tilde{l}_d), \\
q_{cocos\_out} &= \tilde{\sigma}_{Ip}\, \tilde{\sigma}_{B_0}\, \tilde{\sigma}_{\rho\theta\varphi}\, q_{cocos\_in}.
\end{aligned}
\tag{47}
$$

These relations allow a general transformation for the three kinds of transformation discussed at the beginning of this Appendix. The sign of I_p and/or B_0 in output results from the coordinate conventions transformation or can be specified explicitly. Similarly, the transformation with different assumptions for the normalization, for example "si" on one hand and "normalized" on the other hand can be obtained as well. As an example, Eq. (15) is recovered by setting $cocos\_in = 2$, $\sigma_{Ip,in} = \sigma_{B0,in} = 1$, $l_{d,in} = 1$, $l_{B,in} = 1$ and $e_{\mu0,in} = 0$ corresponding to the CHEASE assumptions, and taking any cocos and si units in output.

Similarly, Eq. (21) is obtained from Eq. (47) by setting $cocos\_out = 2$, $\sigma_{Ip,out} = \sigma_{B0,out} = 1$, $l_{d,out} = 1$, $l_{B,out} = 1$ and $e_{\mu0,out} = 0$ since we want the CHEASE assumptions in output.

Note that plasma parameters might be related to a given sign convention of the coordinate systems. For example the toroidal and poloidal rotation should be positive in the direction of φ and θ respectively. Thus if the direction of φ changes, the effective sign of v_φ should change as well, following $\tilde{\sigma}_{R\varphi Z}$. Since the effective direction of θ depends on $\sigma_{R\varphi Z,cocos}\,\sigma_{\rho\theta\varphi}$, we have:

$$\begin{aligned} v_{\varphi,out} &= \tilde{\sigma}_{R\varphi Z}\, v_{\varphi,in}, \\ v_{\theta,out} &= \tilde{\sigma}_{R\varphi Z}\, \tilde{\sigma}_{\rho\theta\varphi}\, v_{\theta,in}. \end{aligned} \tag{48}$$